LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2023**

UST 2501 – CONTINUOUS DISTRIBUTIONS

Date: 29-04-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION – A

Answer ALL the questions **(10 x 2 = 20)**

1. Define two dimensional random variable.
2. A variable X is distributed at random between the values 0 and 4 and its p.d.f is given by $f(x) = kx^3 (4-x)^2$. Find the value of k.
3. Write the mean and variance of rectangular distribution.
4. If X is uniformly distributed with mean 1 and variance 4/3, find P(X<0).
5. State the differences between normal and standard normal distribution.
6. Write down any two properties of exponential distribution.
7. Define t-distribution (for single mean).
8. State any two uses of chi-square distribution.
9. Write a short note on F-distribution.
10. Define order statistics.

SECTION – B

Answer any FIVE questions **(5 x 8 = 40)**

11. The joint probability distribution of X and Y is given by the following table:

X \ Y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

Compute (i) The marginal probability distribution of Y.
(ii) The conditional distribution of Y given that X=2.
(iii) The covariance of between X and Y.

(2+3+3)

12. Derive the m.g.f of uniform distribution and hence find its mean and variance.
13. If X is normally distributed and the mean of X is 12 and S.D is 4. Find out the probability of the following: (i) $X \geq 20$ (ii) $X \leq 20$ and (iii) $0 \leq X \leq 12$. (2+2+4)
14. State any eight properties of normal distribution.
15. Derive the p.d.f of single r^{th} order statistic.
16. Derive the r^{th} raw moments of F-distribution and hence find its mean.
17. Write the statement and proof of lack of memory property of exponential distribution.
18. Let X and Y have joint p.d.f is given by: $g(x,y) = \begin{cases} \dfrac{e^{-(x+y)} x^3 y^4}{\Gamma 4 \Gamma 5} ; & x>0,\ y>0 \\ 0 ; & elsewhere \end{cases}$.

Find the p.d.f of $u = \dfrac{X}{X+Y}$.

SECTION – C

Answer any TWO questions **(2 x 20 = 40)**

19 (i) The daily consumption of milk in a city, in excess of 20,000 liters, is approximately distributed as a Gamma variate with parameters a= 1/10000 and λ=2. The city has a daily stock of 30000 liters. What is the probability that the stock is insufficient on a particular day?

(ii) Derive the m.g.f of normal distribution and hence find its mean and variance. (8+12)

20. If two random variables X and Y have the following joint probability density function:

$$f(x, y) = \begin{cases} 2 - x - y & ; 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & elsewhere \end{cases}$$

Find (i) Marginal probability density functions of X and Y.

(ii) Conditional density functions of X given Y=y and Y given X=x.

(iii) Var (X) and Var(Y) and

(iv) Covariance between X and Y. (4+4+6+6)

21. Derive the central moments of t-distribution and hence find its mean and variance.

22. State and prove central limit theorem.